

Introduction to Nonlinear Dynamics: Backpaper
exam
December 2005

Attempt all five questions. Each question is worth 20 points.

1. Consider a 2×2 -matrix A with trace t and determinant d . Determine the conditions in terms of t and d for A to have purely imaginary eigenvalues.
2. Consider the differential equation $\dot{x} = x + \cos(t)$. Prove that there is a unique periodic solution for this equation.
3. Consider a smooth dynamical system $\dot{x} = F(x)$ on the unit disc. Assume that for each point x on the boundary of the unit disc, $F(x)$ is the outward normal vector. Show that the dynamical system has an equilibrium point in the disc.
4. Consider a smooth planar dynamical system $\dot{x} = F(x)$. Suppose there is an embedding of \mathbb{R} in the plane which intersects every orbit exactly once. Show that the dynamical system $\dot{x} = F(x)$ is topologically conjugate to the system $\dot{x} = \hat{i}$.
5. Suppose $f : S^1 \rightarrow S^1$ is an orientation preserving homeomorphism with every orbit dense in S^1 . Show that the rotation number of f is irrational.